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Дискуссии
Discussions

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Deformalization as the immanent part of logical solving

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Abstract: Deformalization is the part of logical process least investigated and studied. It is often non-trivial and hard task because of subjective and objective complexities.

Subjective complexities connected with logic. Deformalization is needed to present results of logical investigations to outsiders. Outsiders usually use languages and formalisms very far from logical ones. Their thesaurus usually barely intersects with logical one. Thus formulations on logical language cannot be appreciated and comprehended by outsiders and formulation of results needs to be completely replaced by non-logical. This task often is like to translating from one natural language into another with radically different semantic structure and system of notions (e.g. from Russian into Chinese and vice versa).

Subjective complexities connected with roles. Systems of values of the problem solver and the decision consumer is radically different. Many aspects which were important during solution are out of scope of interests of the consumer. Many aspects which were "important" for the consumer are to be negligible for the solver but they are to be restored in presentation of the decision. This side of deformalization leads a bridge to the objective complexities.

Objective complexities. Methods applied during formalization and solving induce "dual" methods are to be applied during deformalization.

General conclusions and propositions. After analyzing whole process of logical solving in its unity it is possible to make some conclusions how logic can take a place which it is worth both in scientific analysis and in education.

Interesting in more detailed speculations of this matter are addressed to the Russian variant.

Keywords: Applied logics, formalization, deformalization, translation from logical languages, methodological conclusions, pedagogic conclusions

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1. Logical Process

Let us remember the structure of the process of problem solving by using logics as described in [Nepejvoda, 2017] and main statements made there.

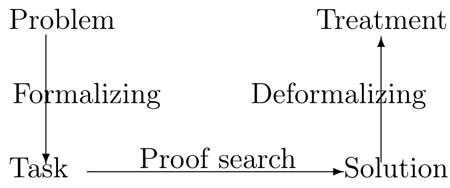

Fig. 1. Solving process

State of arts with deformalization in logical investigations is "almost ignoring". Before to step into this new matter we need to summarize state of art with formalization.

Formalization includes both well described in scholar studies parts and poorly investigated ones.

Omitting of "negligible" or "insignificant" values and properties is well presented and deeply analyzed in physics, applied mathematics, logic and so on. Its philosophical treatment is almost always adequate and often sufficiently deep.

Introduction of higher order notions (abstractions, ideal objects) is also well studied logically and philosophically (even better than this topic for numerical models) though it is often somewhat poorly treated in natural sciences and by common sense.

Approximative models are well and deeply studied in numerical mathematics but in logics their status is poorly described. Logical approximation is purely qualitative and numerical insights are mainly misleading here.

Transformation notions into terms is well studied both for numerical and in logical paradigms of scientific thinking. But terms are very often mixed with initial notions in usual scientific activity, in praxis and in common sense.

2. Non-triviality of deformalization in common settings

Formal solutions are precise (modulo formalization) but not necessarily adequate.

Notions transformed into terms too often can reveal inadequacy of transformations only after solution is found. And sometimes only after it is implemented. Omitted aspects and hidden epistemological assumptions can avenge themselves. It is dangerous because usually they remain hidden in presentation of precise results. Moreover, epistemological assumptions of standard formalism are usually maximally strong (e.g. classical logic as mentioned in [Nepejvoda, 2017], and real numbers, see theory of measurements [Stevens, 1946, Pfangzal, 1973, Roberts, 1985]). Hypotheses necessary to correct application of these notions almost never are satisfied in full scale. Moreover too often such "common" notions are applied when their underlying epistemological assumptions are completely wrong. The well known example here is so called *scientometrics* [Scientometrics, 1978], thoroughly used anywhere in the world to "evaluate and control" scientific activity. Scientometrics is only (maybe striking) an example of current habit to evaluate all by one-dimensional numerical values.

Our formal models are almost always approximative ones. Theory and praxis of numerical approximations had been well developed in XIX century [Kelvin, Tait, 1912] and nowadays it successfully progresses (see e.g. [Anastassiou, Oktay, 2013]. A logical approximation is qualitative one and is poorly studied. Consider an example.

Example 1. "Each human has a mother" is "true" statement and can be formalized logically as

$$\forall x\,(\mathrm{Human}(x) \supset \exists y\,(\mathrm{Human}(y) \wedge \mathrm{Mother}(y, x))) \qquad (1)$$

But how for Adam (or Manu)?

Nevertheless you probably agree that this formalization is good and mention the counterexample of Adam can be reasonable maybe only in formal analysis of Bible.

Thus a formal solution is to be tested for adequacy by completely different and often informal methods. Otherwise hidden assumption and roughening will avenge recklessly. It is better if this vengeance will occur earlier than the decision would be implemented and widely used practically (scientometrics again is here a striking example).

Usually a sufficiently good testing method for mathematical and logical solutions is physics or common sense. This way of "testing and debugging" of formally correct solutions is well shown in the classical treatise [Kelvin, Tait, 1912]. But this treatise also contains examples when this checking do not prevent errors (e.g. see there the computation of the age of Earth and Sun).

3. Absolute subjective aspects

Even if a solution is *in principle* adequate it is often expressed in scientific language and by thesaurus strange for addressee. Unfortunately logical language which is hardly understandable by overwhelming majority of other scientists and practitioners. But there are a lot of less "exotic" examples which can be viewed comparing treatises [Kelvin, Tait, 1912] presenting the classical physics and [Ivanov, 2015] presenting the quantum physics. The same words are used differently.

Usually the logical structure of a complex result in each precise science is the following:

$$A_1 \& \cdots \& A_n \to B_1 \& \cdots \& B_k. \tag{2}$$

Here A_i are conditions or assumptions and B_j are results. Here are some fine moments.

Some of premisses A_i can be hardly understandable and almost non-significant for practitioners.

Example 2. Soviet times. A mathematician explains in all details the results of his work chartered by an agricultural establishment. He have built a statistical models of losses of agricultural products. His report lasts more than half of hour. Charterer is boring and almost sleeping.
Mathematician: "Assume that distribution of losses is normal".
Charterer (slightly awakened): "Of course, it is not insane".

The mathematician noticed and heard nothing. He continued his speech.

Some of results also can be completely blah for addressee (e.g. that our functions are smooth and belong to space $\mathbb{C}_2$). They can be omitted. But it is fair to hide them into comments, e.g.:
By statistical methods we showed...
Proposed mathematical methods can be correctly applied here.

And last by not least result is to be edited to present it in more attractive manner without falsifying. Below we consider some aspects of editing. Now we concentrate on logical languages and results.

4. Aspects of deformalization of logical results

There are the following main aspects of deformalization activity.

1. Treatment of the general structure of proposition.

2. Treatment of quantifiers and embedded quantifiers.

3. Treatment of logical connectives.

4. Editing of deformalization result.

4.1. Treatment of the general structure

Propositions are translated by blocks. These blocks can be sufficiently large and complex as in modern systems of machine translation (Translation memory, [Rlimam, 2007, Lagoudaki, 2006, Pym, 2013]).

Example 3. If word by word translate between very different by their logic languages we can get things like the following. There are two sentences expressing the same fact.

Vanya has a hat

У Вани есть шляпа

After word for word translation we get

(English $\to$ Russian $\to$ English)

Vanya fucks a hat

Russian $\to$ English

Near Vanya exists a hat

Clear, monosemantic and univocal structure of logical languages makes possible to divide the complex proposition into several simple ones. Nevertheless in logic it is possible to write clumsy and badly comprehensible formulas. The situation here is like to situation in well designed programming language. Programmers understand that it is necessary to write not only syntactically correct but well structured and well commented programs. Now we try to describe "well structured" formulas for usual predicate language. Note that usually treated as standard normal forms (prenex or disjuncts particularly) convenient for some theoretical considerations are inconvenient for juggling between formal and informal and for clear and expressive formal description.

Simplest blocks in many logics are aristotelian ones: $\forall x\,(A(x) \to B(x))$, $\exists x\,(A(x)\&B(x))$. Formalizing natural sentences in predicate calculus we can restrict yourself (without "loss of generality", not increasing seriously length of statements and making simpler their understanding and transformations) by conjunctions of *polished* formulas of the following structure.

Definition 1. a) Elementary formulas and their negations are superpolished and polished.

b) Conjunction of superpolished formulas is superpolished and polished.

c) Disjunction and negation of superpolished formulas is polished.

d) Formulas of the form $\forall x\,(A(x \to B(x))$, $\exists x\,(A(x)\&B(x))$, where A is superpolished and B is polished, are superpolished.

Of course in non-classical logic this definition needs some polishing. It works well in classical and intuitionistic logics. We consider below deformalization of modal propositional and polished predicate formulas.

4.2. Treatment of quantifiers and embedded quantifiers

Transliteration of quantifier structure usually leads to awkward natural language statements. Moreover resulting statements looks unnatural and macaronic mix of natural and debris of formal languages. To overcome this it necessary to apply some fantasy and art.

Example 4. Consider an example of practical modal sentence used in [Nepejvoda, 2018].

$$\mathbf{AG}(Req \to \mathbf{AF}Ack) \tag{3}$$

It can be read as

"*For each request there will be acknowledgement*"

This is precise, is straightforward but a bit clumsy.

"*Each request will be processed*"

is better.

Now consider an example how from many variants how to read a propositional connective select an adequate.

$$\text{Outlaw(Robin Hood)\&Good-Hearted(Robin Hood)} \tag{4}$$

Here a variant

"*Robin Hood was an outlaw and good hearted*"

seems adequate maybe only in a dispute when somebody defends a proposition that criminals cannot be good people and you oppose him/her. A variant "*Robin Hood was good-hearted despite he was outlaw*"

is of course more adequate and better.

Embedded quantifiers can be often replaced by actions when they describe a constructive statement/

Example 5. Consider the following statement.

$$\forall x(x \in \mathbb{N} \to \exists y(y > x\&\text{Prime}(y))) \tag{5}$$

Its "transliteration" is barely understandable and maybe monstrous.

For each x if x belongs to the set of natural numbers then exists y such that yis greater than x and y is a prime number.

There is a fine constructive translation.

"*For each number can be found a larger prime number*"

Another fine translations needs applying some context and extra knowledge.

There are arbitrary large prime numbers.

There is infinitely many prime numbers.

We can see that finding a good variant often demands understanding an unmanifest context and using extra knowledge (as in translation between natural languages).

We see that in good translation of quantified sentences no bounded variable is mentioned explicitly. And let us note that we also stepped into editing of deformalized statements.

4.3. Treatment of logical connectives

Each logical connective has many variants how to be expressed in natural language because here the same term represents different notions. We are to choose a convenient in the particular situation.

Example 6. Consider an example of practical modal sentence used in [Nepejvoda, 2018].

$$\mathbf{AG}(Req \to \mathbf{AF}Ack) \tag{6}$$

It can be read as

"*For each request there will be acknowledgement*"

This is precise, is straightforward but a bit clumsy.

"*Each request will be processed*"

is better.

Now consider an example how from many variants how to read a propositional connective select an adequate.

$$\text{Outlaw(Robin Hood)\&Good-Hearted(Robin Hood)} \tag{7}$$

Here a variant

"*Robin Hood was an outlaw and good hearted*"

seems adequate maybe only in dispute when somebody defends a proposition that criminals cannot be good people and you oppose him/her. A variant
"*Robin Hood was good-hearted despite he was outlaw*"
is of course more adequate and better.

4.4. Editing of deformalization result

We have considered this during examining other aspects of deformalization.

4.5. Brief summary

Logical languages are principally different from natural or programming languages. Thus any direct translation become at least clumsy and vague. We are to use context, extra knowledge and common sense to make them clear and really more adequate.

So, after getting the result we have two new tasks.

1. To comprehend it.
2. To explain it to others having another paradigm and do not mastering logical languages.

Both tasks can be non-trivial.

5. Final

5.1. Aesthetic, effectiveness and adequacy

Each statement can be translated into different language by several ways and no one is completely precise. Attempts to reach full precision failed. A striking example is here a sophisticated project UNL (Unified Networking Language) [AGDA, 2017, Uchida et al, 2005, Martins, 2013]. This project is intended to be global. The system "key, modifiers and relations" pretends to express precisely meaning of each word and peculiarities of phrases. Key (English root) describes an approximate meaning of a concept; modifiers try to stress distinctions from English semantics, relations describe the structure of a sentence and role of a notion in the sentence.

Example 7. Маша вышла замуж.
agt(marry(icl>do,icl>woman,icl>time(past):02),
Masha(icl>person,icl>woman),ptn(man):01)
This is described for foreigner also a "hidden" semantics if the Russian sentence:
"The woman Masha have made marriage with a man"
Direct translation "Masha have married" lost many obvious for Russian attributes.

Of course universal language become non-universal. Book [Fomichov, 2009, p. 140] says:

Fact 1. First of all, the language UNL is oriented at representing the contents of only separate sentences but not arbitrary discourses. Even the UNL specifications published in 2006 don't contain a theory of representing the meanings of discourses.

Thus very ambitious project ("A gift for Millenium") lead to cumbersome but nevertheless not fully precise descriptions. This was one more exemplification of Arnold's principle [Arnold, 2004]. Full precision is inaccessible for complex situations. A good formalism is to be deliberately imprecise. It is to be adequate only in its main domain. Less precise model usually can be made better. Too precise one is incurable. It is described by the Russian proverb (I don't know a good English analogue):

Недосол на столе, пересол на спине.

Of course if a model is applied outside its domain of adequacy troubles would begin. If the author(s) claimed their model as "universal" one they can be responsible for those troubles. Otherwise only too positive thinking and optimistic applicant does.

Thus precision is the secondary from important aspects. Adequacy and often even aesthetical criteria are more crucial. Adequacy cannot be mixed with complete infallibility. Often it is better to have a small chance of error than move too slow or into a global deadlock. By the way only a fine decision can be sometimes applied beyond its initial domain.

6. Consequences and propositions

Formalization, deformalization and solving are to be three equally important main components of reasoning in Applied logic and in System approach. They are to be considered in complex. Thus it is bad to start from formulation of task and finish by formal result. It is better to start for an imprecise but informative formulation and finish by natural language explanation of the result. Viewing logic in this manner it is reasonable to use thoroughly assistance of proof assistant programs like AGDA [AGDA, 2017] notwithstanding their big shortcomings [Meshveliani, 2017].

Traditional courses of system theory, system approach et al. lead to the other deadlock: try to estimate all by numbers and to overestimate one dimensional model of values (money or testing score) and linear models of "reality". This deadlock is made more deep by habits of people with physical "scientific and materialistic" paradigm of thinking to take into account only those values which can be measured by numbers. As a consequents of this it is usual to present systemless approach and narrow-minded restricted by one dimension approaches as all possible alternatives to system one. This is valid if we consider mathematics as the background of all possible exact argumentations.

But there is another background very poorly consistent with numbers[1] and arrowed to qualitative analysis of notions. This is logic as had been shown in [Nepejvoda, 2008][2] This is not a rival of system approach, this is sight from other side of notions.

Thus logic (in the form of Applied Logic; see e.g. [Kohen, Nagel, 1993] or [Nepejvoda, 2000] as more current approach) but not in forms of mathematical, philosophical or formal logic) can be another background of exact knowledge

[1] This can be seen examining "fuzzy logics"

[2] Words "in mathematical descriptions" were inserted by demand of publishers to make this paper a bit less radical.

especially useful when studying artificial and virtual objects[3]. 20 years experiment in Udmurt State University showed that this can people to enter rapidly into completely new domains of knowledge and practice and solve problems *without deep studying particular domains* [Nepejvoda, 2008]. This they can work with projects in completely different branches of human activity, engineering and science as informational analytics of highest qualification. Applied logic become the connector between mathematical, technical and humanitarian branches of knowledge and tend to connect them into a system of knowledge (instead of heap of data and algorithms). It is very actual because without system a wave of information leads to degradation of mind down to twitter thinking.

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[3] And maybe our World if it is considered as created by the plan of Highest Essence.

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